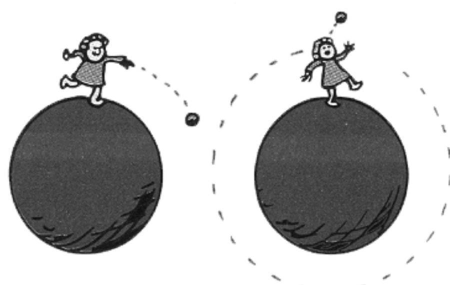
Low Earth Orbit

Newton's Thought Experiment

If you drop a stone, it falls in a straight-line path to the ground below. If you throw the stone horizontally, it will follow a curved path to the ground. If you throw the stone faster, it will land farther away, and the curvature of the path will be less pronounced.

Q. What would happen if the curvature of the path matched the curvature of Earth?

A. In the absence of air resistance, the stone would move in a circular path around Earth.



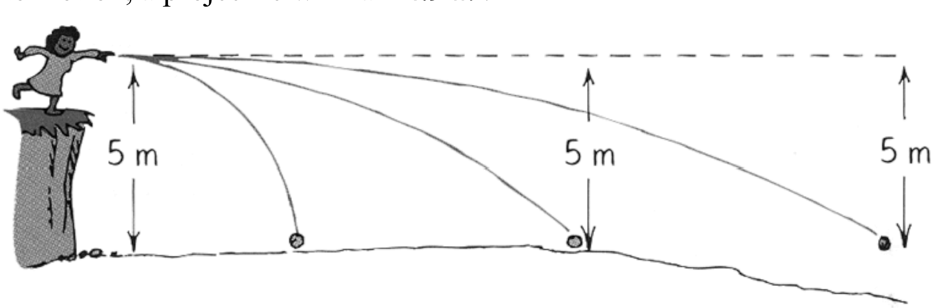
Q. How fast would the stone have to be thrown horizontally for it to orbit Earth?

A. It depends on the rate at which the stone falls and the rate at which Earth curves.

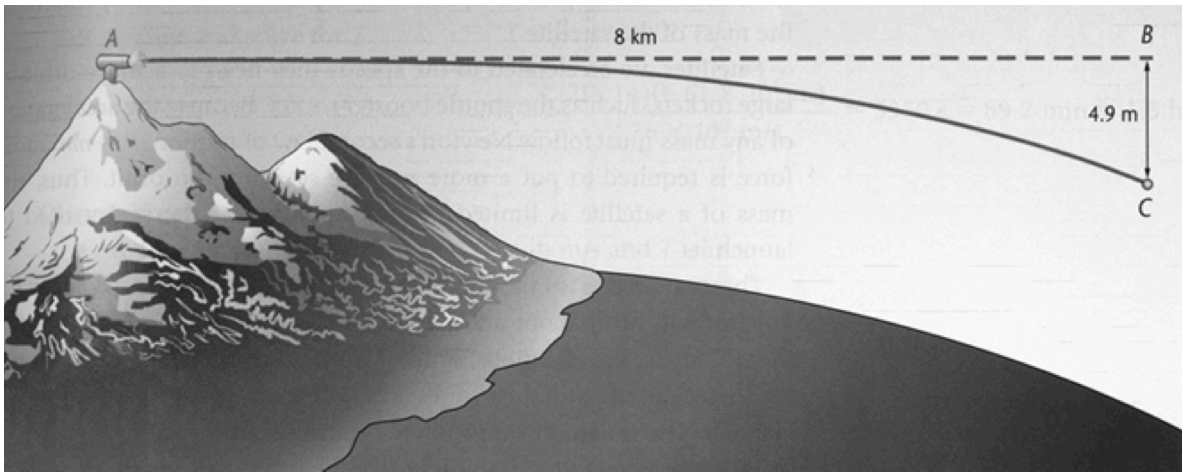
Recall that a stone dropped from rest will accelerate $9.8\ m/s^2$. How far will the stone fall in one second?

$$\begin{aligned} d &= v_i t + \frac{1}{2}at^2 \\ &= (0)(1) + \frac{1}{2}(9.8)(1)^2 \\ &= 4.9\ m \end{aligned}$$

Also recall that the same is true of any horizontally launched projectile. That is, in the first second of its motion, a projectile will fall $4.9\ m$.



It is a geometric fact that the curvature of Earth is such that its surface drops a vertical distance of $4.9\ m$ for every $8000\ m$ tangent to its surface. Thus, if a stone could be thrown fast enough to travel $8.0\ km$ horizontally during the time (1 second) it takes to fall $4.9\ m$, it would follow the curvature of Earth (i.e. it would orbit Earth).



So, we can see that the orbital speed for low Earth orbit is approximately $8\ km/s$, or $29000\ km/h$.

Motion of Planets and Satellites

A satellite is put into orbit by accelerating it to a sufficiently high tangential speed (about $8\ km/s$**)** with the use of rockets. If the speed is too high, the spacecraft will escape Earth's gravity and hurtle off into space. If the speed is too low, the spacecraft will fall back to Earth.

A satellite in an orbit that is always the same height above Earth moves with uniform circular motion. The centripetal force that moves the satellite in a circular orbit is provided by gravity. Therefore,

$$F_c = F_g$$

$$\frac{mv^2}{r} = \frac{GM_E m}{r^2}$$

$$v^2 = \frac{GM_E}{r}$$

$$v = \sqrt{\frac{GM_E}{r}}$$

Speed of an Object in Circular Orbit

Note:

1. We can use this result to find the orbital period of a satellite, since $v = \frac{2\pi r}{T}$.

2. This result can be used for any object in orbit around another. The mass of the central body would replace M_E in the equation.

3. r is not the radius of Earth. It is Earth's radius plus the altitude of the satellite's orbit.

Example 1

A satellite in low Earth orbit is $225\ km$ above the surface. What is its orbital velocity?

Homework

Low Earth Orbit Worksheet #1

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1. Calculate the velocity that a satellite launched horizontally must have in order to orbit Earth, 150 km above the surface. ($7.81\times10^{3}\ m/s$)

2. How long would it take the satellite in question 1 to orbit the earth once? ($5.24\times10^{3}\ s$)

3. Use the data for Mercury in the Useful Planetary Data table to find
   a. the speed of a satellite in orbit 265 km above Mercury's surface. ($2.78\times10^{3}\ m/s$)
   b. the orbital period of the satellite. ($6.41\times10^{3}\ s$)

4. Find the speed with which Mercury moves around the sun. ($4.79\times10^{4}\ m/s$)

5. Find the velocity with which Saturn moves around the sun. Comment on whether it makes sense that Mercury is named after a speedy messenger of the gods, while Saturn is named after the father of Jupiter. ($9.63\times10^{3}\ m/s$)

6. We can consider the sun to be a satellite of our galaxy, the Milky Way. The sun revolves around the center of the galaxy with a radius of $2.2\times10^{20}\ m$. The period of one revolution is $2.5\times10^{8}\ years$.
   a. Find the mass of the Milky Way galaxy. ($1.0\times10^{41}\ kg$)
   b. Assuming the average star in the galaxy has the mass of the sun, find the number of stars in the Milky Way. (5.0×10^{10})
   c. Find the speed with which the sun moves around the center of the galaxy. ($1.7\times10^{5}\ m/s$)

7. Communications satellites are placed in orbit so that they remain stationary relative to a specific area on Earth's surface. They are given the name **geosynchronous satellites** because, to maintain such a position, their period as they orbit must be the same as Earth's (i.e. 1 day). At what height does such a satellite orbit, measured from
   a. the center of Earth? ($4.2\times10^{7}\ m$)
   b. the surface of Earth? ($3.6\times10^{7}\ m$)